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EFFECT OF POROUS MEDIUM AND MAGNETIC FIELD ON THE UNSTEADY FLOW OF DUSTY INCOMPRESSIBLE SECOND ORDER OLDROYD VISCO-ELASTIC LIQUID THROUGH THE RIGHT CIRCULAR CYLINDER

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Abstract

The purpose of the present paper is to analyse the effect of porous medium and uniform magnetic field applied perpendicularly to the unsteady flow of dusty incompressible Oldroyd visco-elastic liquid of second order under the influence of transient pressure gradient through a long right circular cylinder. This problem has been solved in the generalized visco-elastic model and the velocity field for visco-elastic liquid and the dust particles have been derived analytically in the closed form. The particular cases corresponding to Oldroyd, Maxwell, Rivlin-Ericksen dusty liquid and ordinary viscous dusty fluid models are derived for velocity field. There is also the case when uniform magnetic field is withdrawn has been deduced.

INTRODUCTION

The interest in problems of mechanics of system with more than one phase has developed rapidly in the past few years. The situations, which occur frequently, are concerned with the flow of a liquid or gas which contains uniformly distribution of solid particles. Such situations arise, for instance the movement of dust laden air, in fluidization, in the use of dust in gas cooling system, in hydro cyclones, in problems of pollution, in tidal waves etc. The mathematical description of such diverse systems has been discussed very widely. Model equations describing the motion of such mixed system have been given by Saffman[11].

There is another class of flow problems which concerns with the study of the flow of the dusty visco-elastic liquids such as latex particles in emulsion paints, reinforcing particles in polymer melts and rock crystals in molten lava etc. However, the studies of this class of problems and rheological aspects of such flow have not received much attention; although this has become bearing on the problems of petroleum and chemical engineering interest. The unsteady flow of dusty visco-elastic liquids of various kinds through channels of various cross-section with time dependent pressure gradient have been studied by many researchers such as : Dube and Srivastava[4]; Bagchi and Maiti[2]; Kumar and Singh[7] ; Garg, Shrivastava and Singh[5]; Johri and Gupta[6]; Kundu and Sengupta[8]; Dass[3]; Singh, Gupta and Varshney[13]; Varshney and Singh[15]; Singh[14]; Agrawal, Agrawal and Varshney[1]; Singh and Varshney[12]; Prasad, Nagaich and Varshney[10]; Mishra, Kumar and Singh[9]; Tripathi, Sharma and Singh[16] etc. have discussed the effect of magnetic field on the flow of dusty incompressible visco-elastic second order Oldroyd fluid through a rectangular channel.

In the present paper, there is an aim to discuss the unsteady flow of second order Oldroyd visco-elastic liquid through porous medium in a long right circular cylinder under the influence of uniform magnetic field applied perpendicularly to the flow of visco-elastic liquid with transient pressure gradient. The analytical solutions for velocity of visco-elastic liquid and the dust particles are obtained in eligent form. The particular cases for dusty visco-elastic Oldroyd (1958) model liquid, dusty visco-elastic Maxwell liquid, dusty Rivlin-Ericksen liquid, dusty viscous liquid have been derived. There is also the case when the magnetic field is withdrawn which has been deduced.

BASIC THEORY FOR SECOND ORDER OLDROYD VISCO-ELASTIC LIQUID

For slow motion, the rheological equations for second order Oldroyd visco-elastic liquid are:

$$\left.\begin{aligned} &\tau_{ij} = -p\delta_{ij} + \tau'_{ij} \\ &\left(1 + \lambda_1 \frac{\partial}{\partial t} + \lambda_2 \frac{\partial^2}{\partial t^2}\right)\tau'_{ij} = 2\mu\left(1 + \mu_1 \frac{\partial}{\partial t} + \mu_2 \frac{\partial^2}{\partial t^2}\right)e_{ij} \quad \dots (1) \\ &e_{ij} = \frac{1}{2}(v_{i,j} + v_{j,i}) \end{aligned}\right\}$$

where τ_{ij} is the stress tensor, τ'_{ij} the deviatoric stress tensor, e_{ij} the rate of strain tensor , p the fluid pressure , λ_1 the stress relaxation time parameter, μ_1 the strain rate retardation time parameter, λ_2 the material constant , μ_2 the material constant, δ_{ij} the metric tensor, μ the coefficient of viscosity and v_i is the velocity components.

FORMULATION OF THE PROBLEM

Following assumptions have been considered for the equations of motion;

1. The interaction between particles themselves has not been considered.
2. Throughout the motion, density of the dust particles is taken to be constant.
3. The temperature within particles is considered as uniform.
4. The boundary force is neglected.
5. The dust particles are non-conducting and uniform spherical in small size.
6. Mass transfer, radiation and chemical reaction between particles and liquid are not consideration.
7. The effects of the induced magnetic field and the electric field produced by the motion of selectrically conducting visco-elastic liquid are negligible.

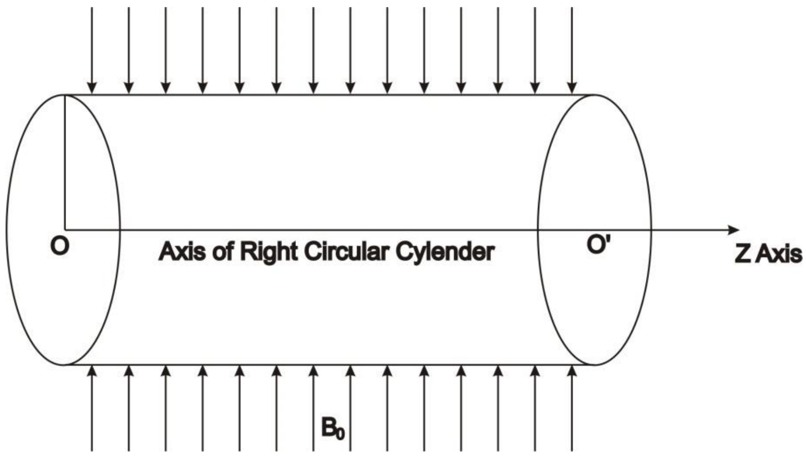

Fig: Schematic diagram of dusty visco-elastic fluid flow in a right circular cylinder.

Let $P(r,\theta,z)$ be the cylindrical polar coordinates in a right circular cylinder of radius a and if (u_r,u_θ,u_z) and (v_r,v_θ,v_z) are the velocity components of the liquid and dust particles respectively at point P. Consider the flow of dusty visco-elastic liquid through a long right circular cylinder of radius a in the direction of z-axis i.e. along the axis of the channel, therefore

$$\left.\begin{aligned} u_r = 0,\ u_\theta = 0,\ u_z = u_z(r,t) \\ v_r = 0,\ v_\theta = 0,\ v_z = v_z(r,t) \end{aligned}\right\} \qquad \ldots(2)$$

Following Saffman (1962), the equations of motion for a dusty second order visco-elastic Oldroyd liquid through porous medium under the influence of transverse magnetic field are:

$$\left(1+\lambda_1\frac{\partial}{\partial t}+\lambda_2\frac{\partial^2}{\partial t^2}\right)\frac{\partial u_z}{\partial t}=-\frac{1}{\rho}\left(1+\lambda_1\frac{\partial}{\partial t}+\lambda_2\frac{\partial^2}{\partial t^2}\right)\frac{\partial p}{\partial z}$$
$$+\upsilon\left(1+\mu_1\frac{\partial}{\partial t}+\mu_2\frac{\partial^2}{\partial t^2}\right)\left(\frac{\partial^2 u_z}{\partial r^2}+\frac{1}{r}\frac{\partial u_z}{\partial r}\right)$$
$$-\left(\frac{\sigma B_0^2}{\rho}+\frac{1}{K}\right)\left(1+\lambda_1\frac{\partial}{\partial t}+\lambda_2\frac{\partial^2}{\partial t^2}\right)u_z+\frac{kN_0}{\rho}\left(1+\lambda_1\frac{\partial}{\partial t}+\lambda_2\frac{\partial^2}{\partial t^2}\right)(v_z-u_z) \quad \dots(3)$$

$$m\frac{\partial v_z}{\partial t}=k(u_z-v_z) \quad \dots(4)$$

where ρ is the density of liquid, *m* the mass of particle, *k* the stokes resistance coefficient, K the permeability coefficient, N_0 the number of density of particles, σ the conductivity of the liquid and B_0 is the intensity of magnetic field.

The boundary conditions for liquid and dust particles are:

$$\left.\begin{array}{lll} u_z=0, & v_z=0, & \text{at } r=a \\ u_z=\text{finite}, & v_z=\text{finite}, & \text{at } r=0 \end{array}\right\} \quad \dots(5)$$

Introducing the following non-dimensional quantities:

$$u^*=\frac{a}{\nu}u_z\,,\ \ v^*=\frac{a}{\nu}v_z\,,\ \ p^*=\frac{a^2}{\rho\nu^2}p\,,\ \ t^*=\frac{\nu}{a^2}t\,,\ \ r^*=\frac{r}{a}\,,$$
$$z^*=\frac{z}{a}\,,\ \ \lambda_1^*=\frac{\nu}{a^2}\lambda_1\,,\ \ \mu_1^*=\frac{\nu}{a^2}\mu_1\,,\ \ \lambda_2^*=\frac{\nu^2}{a^4}\lambda_2\,,\ \ \mu_2^*=\frac{\nu^2}{a^4}\mu_2$$

in the equations (3), (4) and boundary conditions given by equation (5), it is found (after dropping the stars)

$$\left(1+\lambda_1\frac{\partial}{\partial t}+\lambda_2\frac{\partial^2}{\partial t^2}\right)\frac{\partial u}{\partial t}=-\left(1+\lambda_1\frac{\partial}{\partial t}+\lambda_2\frac{\partial^2}{\partial t^2}\right)\frac{\partial p}{\partial z}$$
$$+\left(1+\mu_1\frac{\partial}{\partial t}+\mu_2\frac{\partial^2}{\partial t^2}\right)\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}\right)-\left(H^2+\frac{1}{K}\right)\left(1+\lambda_1\frac{\partial}{\partial t}+\lambda_2\frac{\partial^2}{\partial t^2}\right)u$$
$$+\beta\left(1+\lambda_1\frac{\partial}{\partial t}+\lambda_2\frac{\partial^2}{\partial t^2}\right)(v-u) \quad \dots(6)$$

$$\frac{\partial v}{\partial t}=\frac{1}{\gamma}(u-v) \quad \dots(7)$$

and boundary conditions are:

$$\left.\begin{array}{lll} u = 0, & v = 0 & \text{at } r = 1 \\ u = \text{finite}, & v = \text{finite} & \text{at } r = 0 \end{array}\right\} \quad \ldots (8)$$

where

$$\beta = \frac{f_0}{\gamma} = \frac{N_0 k a^2}{\rho \nu}, \qquad f_0 = \frac{m N_0}{\rho}, \qquad \gamma = \frac{m \nu}{k a^2} \text{ and}$$

$$H = B_0 a \sqrt{\sigma/\mu} \quad \text{(Hartmann number)}$$

Since a transient pressure gradient $-Pe^{-\omega t}$ varying with time t is applied to the dusty visco-elastic Oldroyd liquid of second order, therefore it may choose the solution of equations (4) and (5) as

$$\left.\begin{array}{l} u = U(r)e^{-\omega t} \\ v = V(r)e^{-\omega t} \end{array}\right\} \quad \ldots (9)$$

The corresponding boundary conditions are:

$$\left.\begin{array}{ll} U = 0, V = 0, & \text{at } r = 1 \\ U = \text{finite}, V = \text{finite}, & \text{at } r = 0 \end{array}\right\} \quad \ldots (10)$$

Putting u and v in equation (7), it is obtained

$$V = \frac{U}{1 - \gamma\omega} \quad \ldots (11)$$

From equation (6) with the help of equations (9) and (11), it is obtained

$$\frac{d^2U}{dr^2} + \frac{1}{r}\frac{dU}{dR} + M^2 U = C \quad \ldots (12)$$

where

$$M = \left[\frac{\left\{\omega(1 - \gamma\omega + \beta\gamma) - \left(H^2 + \frac{1}{K}\right)(1 - \gamma\omega)\right\}(1 - \lambda_1\omega + \lambda_2\omega^2)}{(1 - \gamma\omega)(1 - \mu_1\omega + \mu_2\omega^2)}\right]^{\frac{1}{2}} \quad \ldots (13)$$

and

$$C = \left(\frac{1 - \lambda_1\omega + \lambda_2\omega^2}{1 - \mu_1\omega + \mu_2\omega^2}\right)P \quad \ldots (14)$$

Now, by solving equation (12) with the help of boundary conditions given by equation (10), it is found

$$U(r) = \frac{C}{M^2}\left(\frac{J_0(Mr)}{J_0(M)} - 1\right) \quad \ldots (15)$$

where J_0 is the Bessel's function of zeroth order.

From equations (9) and (15), the velocity of second order Oldroyd visco-elastic liquid is obtained

$$u = \frac{C}{M^2}\left(\frac{J_0(Mr)}{J_0(M)} - 1\right)e^{-\omega t} \quad \dots (16)$$

and from equations (9), (11) and (15), the velocity of dust particles is obtained

$$v = \frac{C}{(1-\gamma\omega)M^2}\left(\frac{J_0(Mr)}{J_0(M)} - 1\right)e^{-\omega t} \quad \dots (17)$$

PARTICULAR CASES

CASE I: If material constants $\lambda_2 = 0$ and $\mu_2 = 0$

Then from equations (16) and (17), there are obtained velocities of Oldroyd visco-elastic liquid and the dust particles respectively, where *M* will be

$$M = \left[\frac{\left\{\omega(1-\gamma\omega+\beta\gamma) - \left(H^2+\frac{1}{K}\right)(1-\gamma\omega)\right\}(1-\lambda_1\omega)}{(1-\gamma\omega)(1-\mu_1\omega)}\right]^{\frac{1}{2}} \quad \dots (18)$$

CASE II: If $\mu_1 = 0, \mu_2 = 0$

Then from equations (16) and (17) there are obtained velocities of Maxwell visco-elastic liquid and the dust particles respectively, where *M* will be

$$M = \left[\frac{\left\{\omega(1-\gamma\omega+\beta\gamma) - \left(H^2+\frac{1}{K}\right)(1-\gamma\omega)\right\}(1-\lambda_1\omega+\lambda_2\omega^2)}{(1-\gamma\omega)}\right]^{\frac{1}{2}} \quad \dots (19)$$

Case III: If $\lambda_1 = 0,\ \lambda_2 = 0,\ \mu_2 = 0$

Then from equations (16) and (17), there are obtained velocities of Rivlin-Ericksen visco-elastic liquid and the dust particles, where M will be

$$M = \left[\frac{\left\{\omega(1-\gamma\omega+\beta\gamma) - \left(H^2+\frac{1}{K}\right)(1-\gamma\omega)\right\}}{(1-\gamma\omega)(1-\mu_1\omega)}\right]^{\frac{1}{2}} \quad \dots (20)$$

Case IV: If $\lambda_1 = 0,\ \lambda_2 = 0,\ \mu_1 = 0,\ \mu_2 = 0$

Then from equations (16) and (17), there are obtained velocities of viscous liquid and the dust particles respectively under the influence of magnetic field, where *M* will be

$$M = \left[\frac{\left\{\omega(1-\gamma\omega+\beta\gamma) - \left(H^2 + \frac{1}{K}\right)(1-\gamma\omega)\right\}}{(1-\gamma\omega)}\right]^{\frac{1}{2}} \qquad \ldots(21)$$

DEDUCTION

If magnetic field is withdrawn i.e. $B_0 = 0$, then all the above results in absence of magnetic field can be obtained with slight change of notation.

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